This document explains how to output two thresholds  and  for a simple case.

Details are available in the following paper:

Dixit, A.K., 1989. Entry and exit decisions under uncertainty. Journal of Political Economy 97(3), 620-638.

The stochastic process is geometric Brownian motion.

In the optimality conditions (Eq. 8-11) in Dixit (1989), there are four decision variables. It is quite difficult to solve this set of equations directly.

This system of equations can be simplified into two equations consisting of only two decision variables. Thus, a two dimensional search can be used to find  and .

Mainly, I provided three Matlab programs:

gm.m is to generate a sample trajectory of Q.

search2d.m is used to do the 2-dimensional search.

SolTwoEquOU.m outputs the difference for each combination of  and . When the differences in two equations (Eq. 49-50 in this document) are close to 0,  and  are optimal.

Note that SolTwoEquOU.m I provided here is for reference only, because in our paper we studied the OU process, which is much more complex to understand for a beginner.

Therefore, I encourage learners to start with Dixit (1989) and go through this document and modify sample codes I provided, in whatever language they are familiar with.

Step 1: Simulate the Q, and we have the stochastic process parameter, such as volatility and growth rate.

See gm.m

Step 2: Insert the stochastic process parameters into  and 

Step 3: Simplify the function of and and insert them into Eq. (8-11) in Dixit’s paper

Then the problem of 4 dimensional search becomes 2 dimensional search. See search2d.m

Here, we assumed ,  and , , 





Where (to simplify, we used to represent  )



We assumed the incremental cost savings function is known (is known) 

So we have 

Then to have the first order derivative with respect to Q 

Due to the reason given by page 626 in Dixit’s paper, can be simplified as :

(here  ) (35)

 (here  ) (36)

Then we solve the “value matching” and “smooth pasting” conditions Eq. (37-40) to get Qh and Ql

 (37) which is the same as Eq.(8) in Dixit’s paper, where 

(38) which is the same as Eq.(9) in Dixit’s paper, where 

(39) which is the same as Eq.(10) in Dixit’s paper

(40) which is the same as Eq.(11) in Dixit’s paper

Inserting Eq. (35-36) to Eq. (37-38), we have

 (41)

 (42)

Inserting the first order derivative with respect to Q of Eq. (35-36) to Eq. (39-40), we have

 (43)

 (44)

We rearrange Eq. (41) to have, and insert it into Eq. (43)



Then we have the relationship between C and Qh:

 (45)

And the relationship between D and Qh:

 (46)

We rearrange Eq. (42) to have , and insert it into Eq. (44)

Then we have the relationship between C and Ql:

 (47)

Then we have the relationship betweenD and Ql:

 (48)

Because C and D are constant parameter, Eq.(45)=Eq. (47), and Eq.(46)=Eq. (48) 

So we have two equations (49) and (50) with variable Qh and Ql, we can set up a two dimensional search to solve variable Qh and Ql.

 (49)

 (50)